Liberian Mathematics Teacher Training Program 2023–2024

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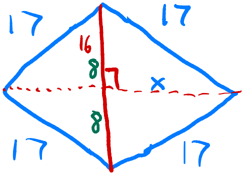
HW Exercise 1

A diagonal of a rhombus has length 16. Each side of the rhombus has length 17. Find the length of the other diagonal.

Let $x = \frac{1}{2}$ the other diagonal.



Pythagorean theorem:

$$17^2 = 8^2 + x^2$$

$$\rightarrow 17^2 - 8^2 = x^2$$

$$\rightarrow 289 - 64 = x^2$$

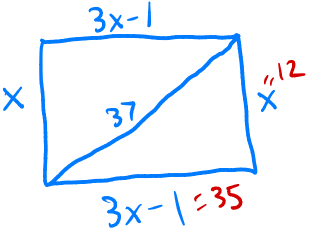
$$\rightarrow 225 = x^2$$

$$\rightarrow 15 = x$$

So diagonal has length: $2x = 2 \cdot 15 = \boxed{30}$

HW Exercise 2

The diagonal of a rectangle has length 37. The length is 1 less than 3 times the width. What is its perimeter? *Hint:* Let x be the width. Then set up an algebraic equation for x. Once you know x, then you know the length as well, and can figure out the perimeter.



Equation: Solve for x

$$37^2 = (3x-1)^2 + x^2$$

$$1369 = 9x^2 - 6x + 1 + x^2$$

$$1369 = 10x^2 - 6x + 1$$

$$0 = 10x^2 - 6x - 1368$$

$$0 = 5x^2 - 3x - 684$$

$$x = \frac{+3 \pm \sqrt{9 + 20 \cdot 684}}{10}$$

$$x = \frac{3 \pm 117}{10}$$

$$x = \frac{3 + 117}{10} = 12$$

$a = 5$
$b = -3$
$c = -684$

Q. Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

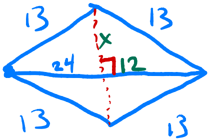
$$\text{Perim.} = 2l + 2w = 2(35) + 2(12) = \boxed{94}$$

HW Exercise 3

Find the area of a rhombus with sides of length 13 and one diagonal of length 24.

Recall: Area of a rhombus

$= \frac{1}{2} \cdot$ (diagonal 1)(diagonal 2)



Solve for x: $x^2 + 12^2 = 13^2$

$\rightarrow x^2 + 144 = 169$

$\rightarrow x^2 = 25$

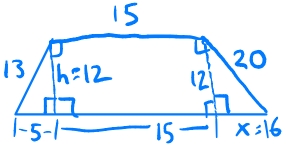
$\rightarrow \boxed{x = 5}$

So other Diagonal $= 2 \cdot \boxed{5} = 10$

Area: $\frac{1}{2} \cdot 10 \cdot 24 = \boxed{120}$

HW Exercise 4

Find the area of the following trapezoid:



Recall: Area $= h\cdot\left(\frac{b_1+b_2}{2}\right)$

h = height. b_1, b_2 are bases (parallel sides).

Know $b_1 = 15$

Exc: $h = 12$ (solve $5^2 + h^2 = 13^2$).

what is b_2? Solve for x: $12^2 + x^2 = 20^2$

$\to 144 + x^2 = 400$

$\to x^2 = 256 \to x = 16$.

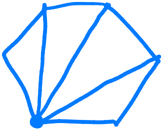
$b_2 = 5 + 15 + 16 = 36$.

Area $= 12\left(\frac{15+36}{2}\right) = 6\cdot(15+36) = \boxed{306}$

Regular polygons

- Recall that a polygon is *regular* if all of its sides are equal and all of its angles are equal.
- We proved earlier that if a regular polygon has n sides, its total interior angle measure is $180(n-2)$ degrees, and so each interior angle measures $180(n-2)/n$ degrees.

Alternate picture $(n=6)$



Divides hexagon into 4 triangles.
Each has 180°
So total angle measure is
$4 \cdot 180^\circ = 720^\circ$.

Nomenclature

Some names for polygons:

Sides	Name	Sides	Name
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	20	Icosagon

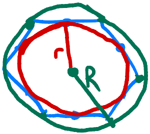
Ex: 147-sided polygon: 147-gon.

Inscribed and circumscribed circles

- Every regular polygon has an *inscribed* and a *circumscribed* circle.
- The radius r of the inscribed circle is the distance from the center of the polygon to the midpoint of any side (it is sometimes called the *inradius* or *apothem*). The radius R of the circumscribed circle (called the *circumradius*) is the distance from the center of the polygon to any vertex.

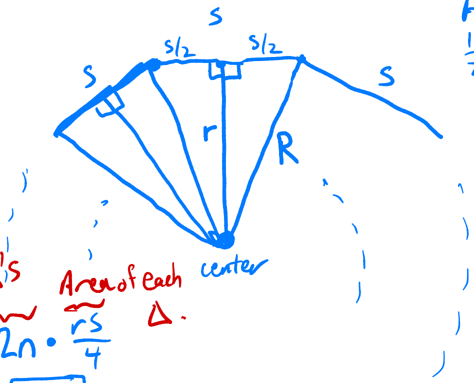
Green: Circumcircle

Red: incircle



Area of a regular polygon

- Suppose a regular polygon has n sides, each with length s, inradius r, and circumradius R.
- Let us calculate the area of the regular polygon by dividing it into triangles.



Area of triangle:

$\frac{1}{2}bh = \frac{1}{2}r \cdot \frac{s}{2}$

$= \frac{rs}{4}$

The polygon comprises $2n$ of these triangles.

of Δ's

Area of each Δ.

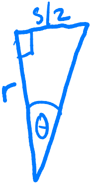
Total area: $2n \cdot \frac{rs}{4}$

$= \boxed{\frac{nrs}{2}}$

Area of a regular polygon, continued

What if we want an area formula only in terms of s and n? We can get this by writing r in terms of s and n.

Goal: Express r in terms of s and n.



Remember: $2n$ triangles make the polygon.

So $\theta = \dfrac{360^\circ}{2n} = \dfrac{180^\circ}{n}$.

Trigonometry: $\tan\theta = \dfrac{s/2}{r}$

OR $\tan\left(\dfrac{180^\circ}{n}\right) = \dfrac{s/2}{r}$

Substitute for r:

$\rightsquigarrow r = \dfrac{s/2}{\tan(180^\circ/n)} = \dfrac{s}{2\tan(180^\circ/n)}$.

$$\text{Area} = \frac{nrs}{2} = \frac{ns}{2} \cdot \overbrace{\frac{s}{2\tan(180^\circ/n)}}^{r} = \boxed{\frac{ns^2}{4\tan(180^\circ/n)}}$$

Area of a regular polygon, examples

- We have seen on the previous slide that the area of a regular polygon with n sides of length s is

$$\frac{ns^2}{4\tan(180^\circ/n)}.$$

$$\frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$$

- Let's calculate this in the case $n = 3$ and $n = 4$.

equilateral triangle square

$$\underline{n=3:} \text{ Area of } \triangle = \frac{3\cdot s^2}{4\cdot\tan(180^\circ/3)} = \frac{3s^2}{4\cdot\tan(60^\circ)} = \frac{3s^2}{4\cdot\sqrt{3}} = \boxed{\frac{\sqrt{3}}{4}\cdot s^2}$$

$$\underline{n=4:} \text{ Area of } \square = \frac{\cancel{4}s^2}{\cancel{4}\tan(180^\circ/4)} = \frac{s^2}{\tan 45^\circ} = \frac{s^2}{1} = \boxed{s^2}$$

Homework Exercises

1. Suppose a regular hexagon has side length s. What is its area?
2. Compare your answer above to the area of an equilateral triangle with side length s. How many times bigger is the hexagon's area? Can you give a geometric reason why this should be true?

$\frac{\sqrt{3}}{4} s^2$.

Thank you for your attention! There will be NO CLASS next week (January 12) because I will be at a conference. On January 19, we will begin our discussion of circles.