Liberian Mathematics Teacher Training Program 2023–2024

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HW Exercise

Two numbers x and y have a geometric mean of 12 and an arithmetic mean of 12.5. Find $x^2 + y^2$.

Recall: Arithmetic Mean: $\frac{x+y}{2} = 12.5$

Geometric Mean: $\sqrt{xy} = 12$

$x+y=25$

$xy = 144$

$y = \frac{144}{x}$

Substitute: $x + \frac{144}{x} = 25$

$\rightarrow \frac{x^2+144}{x} = 25 \rightarrow x^2+144 = 25x$

$\rightarrow x^2 - 25x + 144 = 0$

$\rightarrow (x-9)(x-16) = 0$

So $x=9$ OR $x=16$

Since $y = \frac{144}{x}$, $y=16$ OR $y=9$.

Answer:

$x^2+y^2 = 9^2+16^2 =$

$81+256 = \boxed{337}$

Alternatively:

$x+y=25$

$xy=144$

Want: x^2+y^2

$(x+y)^2=25^2$

$x^2+2xy+y^2=625$

$-2xy \qquad -2\cdot 144$

$x^2+y^2=625-2\cdot 144$

$=\boxed{337}$

Statistics: Measures of dispersion

Even if we know that a data set has a given mean, there is still much we don't know about the data. Some of this information can be gleaned from the so-called "measures of dispersion".

For instance, a data set consisting of the entries

"less dispersed"

$$9.9, 9.9, 9.9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10.1, 10.2$$

is very different from one consisting of the entries

"More dispersed"

$$-10, 1, 10, 24, 25,$$

even though both have mean and median equal to 10.

We will study three measures of dispersion: The *range*, the *mean absolute deviation* and the *standard deviation*.

Most important/Most widely used.

Range

The *range* of a data set is simply the difference between the largest entry and the smallest entry. It is a very crude measure of dispersion. For instance, in the data sets above, the range of the first data set is $10.2 - 9.9 = 0.3$, and the range of the second data set is $25 - -10 = 35$.

$35 > 0.3$, so second data set has a bigger range.

Mean absolute deviation

Given a data set $x_1, x_2, \ldots, x_n$ with mean $\bar{x}$, the *mean absolute deviation* of the set is the average of the numbers $|x_i - \bar{x}|$. That is, the average distance from the mean.

distance from x_i to $\bar{x}$

Example: Mean absolute deviation of the two previous sets

1) 9.9, 9.9, 9.9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10.1, 10.2. $(N=14)$

$\bar{x} = 10$ (Exercise)

Mean absolute deviation: Average of

$$|9.9-10|, |9.9-10|, |9.9-10|, \underbrace{|10-10|, |10-10|, \ldots, |10-10|}_{9 \text{ times}}, |10.1-10|, |10.2-10|.$$

OR Average of 0.1, 0.1, 0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.2

$$= \frac{0.1+0.1+0.1+0+\cdots+0+0.1+0.2}{14} = \frac{0.6}{14} \approx \boxed{0.043}$$

2) −10, 1, 10, 24, 25

$\bar{x} = 10$ (Exercise)

MAD: Average of

$$|-10-10|, |1-10|, |10-10|, |24-10|, |25-10|$$

$$= (20+9+0+14+15)/5 = \boxed{11.6}$$

Standard deviation

The *standard deviation* is used much more often than the mean absolute deviation. It is calculated as follows:
Given a data set $x_1, x_2, \ldots, x_n$ with mean $\bar{x}$, the *standard deviation* of the set is the square root of the average of the numbers $(x_i - \bar{x})^2$. As a mathematical formula, the standard deviation equals

$$\sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}.$$

"Square root of the average of the squared deviations"

Another related quantity is the *variance* of the data set, which is the square of the standard deviation (that is, do the same calulation, but do not take the square root at the end).

VARIANCE = (STANDARD DEVIATION)^2.

Example: Standard deviation of the two previous sets

9.9, 9.9, 9.9, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10.1, 10.2

$\bar{x} = 10.$

1) $x_i - \bar{x}$: -0.1, -0.1, -0.1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.1, 0.2.

1) Write down all deviations from mean

2) $$\frac{(-0.1)^2 + (-0.1)^2 + (-0.1)^2 + \underbrace{0^2 + 0^2 + \ldots + 0^2}_{9 \text{ times}} + (0.1)^2 + (0.2)^2}{14} = \frac{0.08}{14} \approx 0.0057$$

2) Average the squares:

Variance.

3) $\sqrt{0.0057} = \boxed{0.075}$

Standard Deviation

3) Take the square root

$-10, 1, 10, 24, 25.$

$\bar{x} = 10.$

1) $-20, -9, 0, 14, 15$ (deviations)

2) $$\frac{400+81+0+196+225}{5} = \frac{902}{5}$$

$$= \underbrace{180.4}_{\text{variance}}$$

3) $$\sqrt{180.4} = \boxed{13.43}$$

↳ standard deviation

Next week: Deviations (mean absolute, standard) with group data.

Box and whisker plots

- A *box and whisker plot* (or sometimes just "boxplot") is a way of visualizing a data distribution that takes into account both its central tendency and its dispersion.
- It is based on percentiles.
- A box is drawn parallel to an axis (can be horizontal or vertical — will be vertical in our case) stretching from the 25th to the 75th percentile.
- The length of this box is called the "Inter-quartile range", or IQR.
- A horizontal line is also drawn at the 50th percentile, or median of the data.
- "Whiskers" extend above and below the box to the maximum and minimum of the data points, respectively.
- The range and median of the data can be read off directly from the box and whisker plot.

Example

Let us draw a box and whisker plot for the following data: 45, 48, 50, 54, 57, 60, 60, 62, 63, 63, 64, 65, 67, 68, 69, 71, 71, 72, 72, 74, 74, 75, 75, 76, 80, 83, 84, 88, 100, 100, 100.

To draw: Calculate: MAX = 100

MIN = 45

25^{th}% = 62

Median = 50^{th}%. = 71

75^{th}% = 76

N = 31.

25^{th}% rank: 0.25(31+1) = 8 → 62 is 25^{th}%.

50^{th}% rank: 0.5(31+1) = 16 → 71 is 50^{th}%

75^{th}% rank: 0.75(31+1) = 24 → 76 is 75^{th}%.

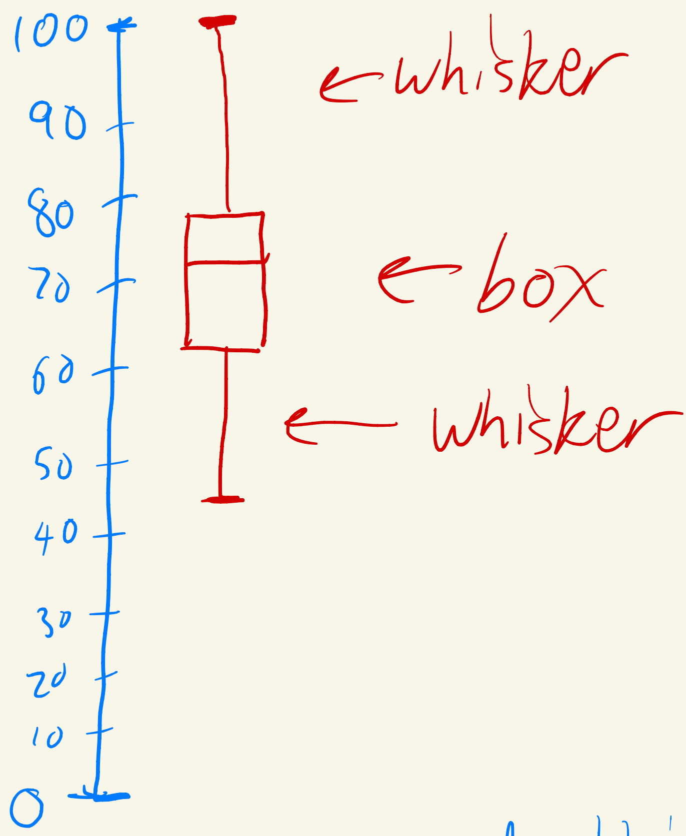
Max = 100

Min = 45

25^{th}% = 62

50^{th}% = 71

75^{th}% = 76.



Next week: show how to do this in EXCEL.

Example, continued

Microsoft Excel can also draw box and whisker plots. Let us use the above data as an example.

NEXT WEEK

Homework Exercises

1. Calculate the range, the mean absolute deviation and the standard deviation of the data set $1, 2, 3, 4, 5, 6$.
2. What happens to the range, the mean absolute deviation, and the standard deviation of a data set if all the entries are increased by 5? What about if all the entries are multiplied by 5?

Thank you for your attention. Next week we will summarize what we have learned about statistics and begin our unit on geometry.

↳ Box and Whisker Plots in Excel

→ Deviations for group data

→ Introduction to plane geometry.