Liberian Mathematics Teacher Training Program 2023–2024

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HW Exercise

Calculate the mean and standard deviation of the group data given by the following frequency table:

Time	Frequency	Class Midpoint (x_i)	$\bar{x}-x_i$	$f_i(\bar{x}-x_i)^2$
5 → 0 to < 10	37	5	19.985	14778
15 → 10 to < 20	123	15	9.985	12263
25 → 20 to < 30	350	25	-.015	0
35 → 30 to < 40	116	35	-10.015	11635
45 → 40 to < 50	40	45	-20.015	16024

First, Calculate Mean of group data.

$$\text{Mean: } \frac{5\cdot 37+15\cdot 123+25\cdot 350+35\cdot 116+45\cdot 40}{37+123+350+116+40} = \frac{16640}{666} = \boxed{24.985}$$

Mean

N

$$\text{Standard deviation} = \sqrt{\frac{14778+12263+0+11635+16024}{666}} = \boxed{9.06}$$

Alternative formula for group standard deviation:

$$\sqrt{\frac{\sum_i f_i x_i^2}{N} - \left(\overline{x}\right)^2}$$

This is equivalent to our other formula.

Advantage: Easier to calculate with.

Disadvantage: Meaning is not as clear.

Recall: Standard deviation of group data

Suppose we are given group data in a frequency table where each row of the table corresponds to a range of values. How do we calculate the standard deviation?

- We have already seen how to calculate the *mean* of the data: We look at the *midpoint* of each class, and take a weighted average of these midpoints, weighted by the frequency of each class.
- Call the mean of the data $\bar{x}$. Let x_i be the midpoint of the ith class and let f_i be the frequency of the ith class. Lastly, let N be the total number of measurements (so N equals the sum of all the f_i).
- The standard deviation can now be calculated using the following formula:

$$\sqrt{\frac{\sum_i f_i(x_i - \bar{x})^2}{N}}.$$

Plane Geometry

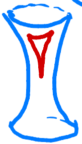
- Plane geometry is the study of geometry in the flat 2-dimensional plane.
- The main quantities we measure in plane geometry are *angles*, *lengths*, and *areas*.

Other kinds of geometry:

- Solid geometry (3-dimensional geometry)
- spherical geometry

← angles of a triangle add to more than 180°

- hyperbolic geometry

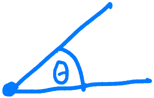


← angles of a triangle add to less than 180°

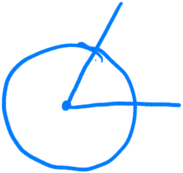
- projective geometry
- higher-dimensional geometry - - - -

Angles

- An angle is formed by two rays or line segments originating from the same point.
- It can be measured in degrees or radians.
- For Euclid, the fundamental angle was the *right angle*, and all other angles were measured in terms of right angles.
- Example: a straight angle.

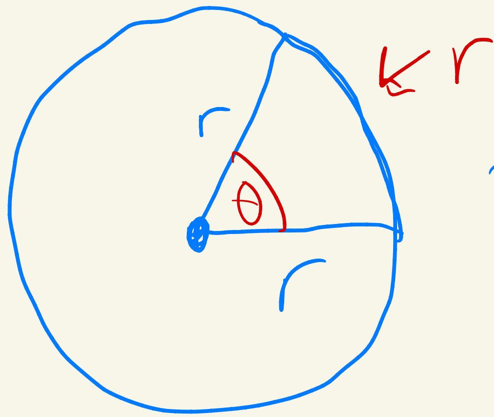


Degrees: A full circle is $360°$. Angles are measured based on what fraction of a circle they cover.



If this is $1/6$ of the circle, the angle measures $60°$.

Radians: An angle that cuts out an arc of a circle with length equal to the circle's radius is 1 radian.

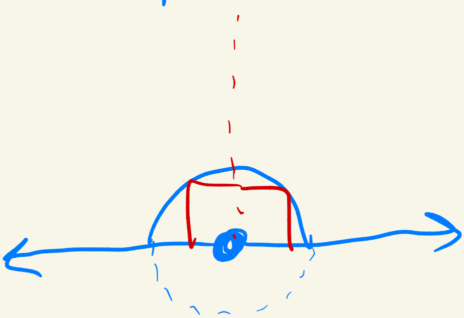


Then $\theta = 1$ radian

In fact: 1 radian $= \frac{180}{\pi}$ degrees

≈ 57.3 degrees.

<u>Ex</u>: Straight angle



Half a circle: 180°

In Euclid's language: "2 right angles"

Radians: $180^\circ \times \dfrac{1 \text{ radian}}{\frac{180}{\pi} \text{ degrees}} = \dfrac{180}{\frac{180}{\pi}}$ radians

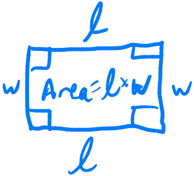
$= \pi$ radians

Lengths

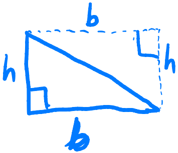
- The *length* of a line segment is the distance from one endpoint to the other.
- In real-world applications of geometry, this can of course be measured in feet, meters, miles, light-years, etc.
- In the pure mathematical study of plane geometry, we generally do not include units when talking about lengths.

Areas

- We define the area of a rectangle to be equal to its length times its width.
- Areas of other figures bounded by straight lines can be figured out by using the fact that if two figures are non-overlapping, then the area of the total figure is the sum of the areas of the individual figures.
- Areas of figures that are not bounded by straight lines are actually pretty hard to compute without using calculus.
- Example: Area of a parallelogram.

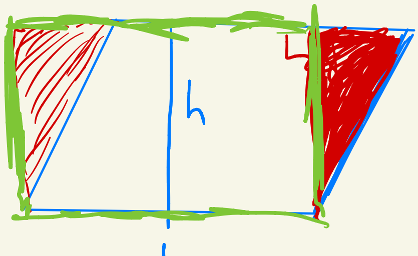


Ex: Right triangle:



The rectangle with area bh is made up of 2 non-overlapping right triangles, so each one must have area $\frac{1}{2}bh$.

Example: Area of a parallelogram



Area of rectangle = area of parallelogram + area of triangle on left − area of triangle on right = area of the parallelogram.

So area of parallelogram equals the area of rectangle, that is,

$\boxed{bh}$.

Approaches to geometry

- There are two main approaches to plane geometry.
- “Synthetic approach” (Euclid)
- “Analytic approach” (Descartes)

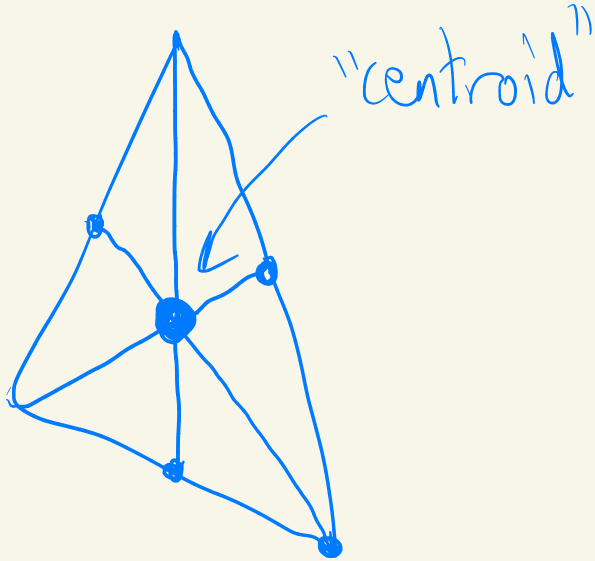
Synthetic approach

- The idea here is to start with some terms that are left undefined (for example: “point”, “line”, “between”) and some basic axioms (for example: “for any two distinct points in the plane, there exists exactly one line passing through both of them”).
- Then, you can work from the ground up, proving all statements of plane geometry from the axioms.
- Even in the synthetic approach, it is useful to draw diagrams to aid in one’s understanding.
- This is the approach of Euclid’s *Elements*.

In spherical geometry axiom above does not hold.

Analytic approach

- In this approach, we think of geometry as taking place in the Cartesian (coordinate) plane.
- This can often simplify proofs, although some might view the proofs as less elegant, since the proofs tend to be more "algebraic" rather than "geometric".
- For instance, consider the statement "The three medians of a triangle all meet at a point" (recall that a *median* is a line segment linking a vertex of a triangle to the midpoint of the opposite side).
- An analytic proof would proceed by writing down the three vertices of the triangle as points in the plane (say, (a, b), (c, d), and (e, f)), then computing the equations of the three medians (as lines), and then showing that there is a point lying on all three medians using the equations.
- A synthetic proof would rely on drawing extra lines, and using theorems about parallel lines, similar triangles, etc.


"centroid"

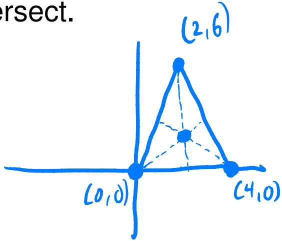
This semester

In general, our emphasis this semester will not be on proofs, but rather on formulas and their derivations. For instance, we will derive the area formula for a regular polygon (not necessarily a triangle or a square).

Homework Exercise

Suppose a triangle has vertices at $(0, 0)$, $(4, 0)$, and $(2, 6)$.

- Compute equations for the three medians of the triangle.
- Solve these equations simultaneously to find the point where the three medians intersect.



There is NO CLASS next week. The week after we will discuss triangles in more depth.