Liberian Mathematics Teacher Training Program 2023–2024

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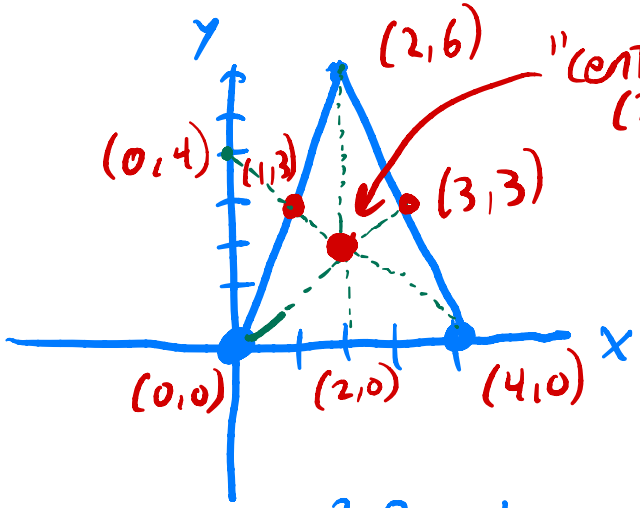
HW Exercise

Suppose a triangle has vertices at $(0,0)$, $(4,0)$, and $(2,6)$.

- Compute equations for the three medians of the triangle.
- Solve these equations simultaneously to find the point where the three medians intersect.



$\frac{(2,6)+(4,0)}{2} = \frac{(6,6)}{2} = (3,3)$ ← midpoint on right

$\frac{(0,0)+(2,6)}{2} = \frac{(2,6)}{2} = (1,3)$ ← midpoint on left

→ $(0,0)$ to $(3,3)$: Slope $\frac{3-0}{3-0} = 1$

y-intercept: $y=0$

$y = mx+b$

slope, y-intercept

Line equations:

1) $x=2$
2) $y=x$
3) $y=-x+4$

Solve: $(2,2)$

Check: $2=-2+4$ ✓

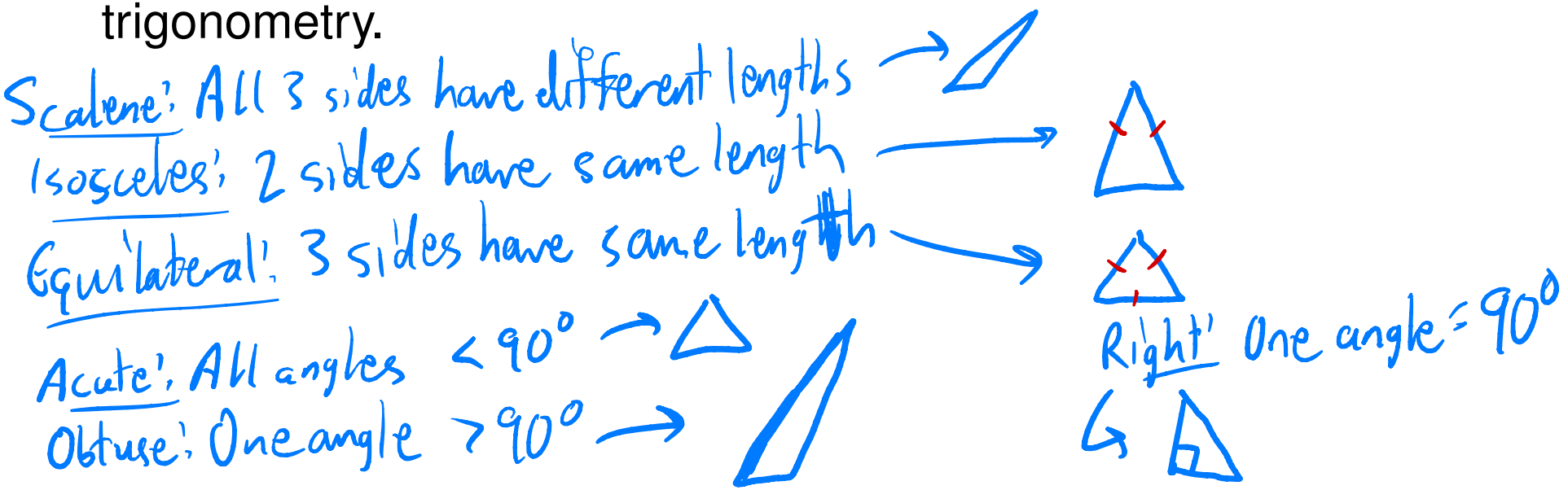
$(4,0)$ to $(1,3)$

slope: $\frac{3-0}{1-4} = -1$

y-intercept: 4

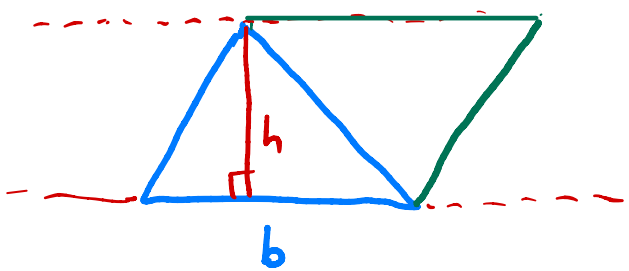
Triangles

- A *triangle* is the simplest type of polygon. It has three sides and three angles.
- There are various ways of dividing triangles into categories:
  - Scalene, isosceles, and equilateral
  - Acute, obtuse, and right
- The study of triangles is very rich, and we will only scratch the surface this semester. In particular, we will not do any trigonometry.



Area of a triangle

- The fomula for the area of a triangle is $\frac{1}{2}bh$, where b is the *base* of a triangle (the length of a side) and h is the *height* (the distance from the base to a parallel line passing through the opposite vertex).
- Why is this true? Recall that we showed last time that the area of a parallelogram is bh, where b is the base (the length of one side) and h is the the height (distance from the base to the other parallel side).
- From this, we can derive the area formula for a triangle.

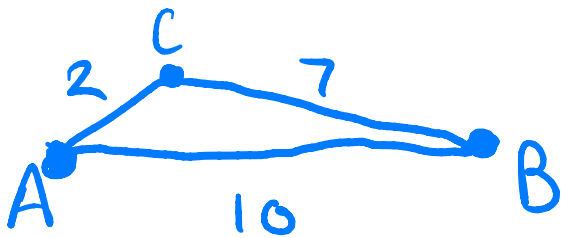


2 congruent triangles make a parallelogram with area bh. This means that each of the triangles has area $\frac{1}{2}bh$.

The triangle inequality

1. Does there exist a triangle with side lengths 2, 7, and 10?
2. If a triangle has two sides of length 7 and 13 then what are the possibilities for the length of the third side?

1)



A to B : 10

A to C to B : 2 + 7 = 9

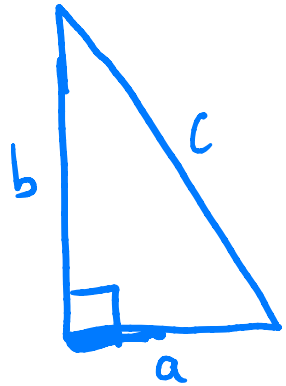
Conclusion: This triangle does not exist! Because it violates the principle that the shortest distance between 2 points is a straight line.

Triangle Inequality: In a triangle, the sum of any two side lengths is always greater than the third side length.

The Pythagorean theorem

- The Pythagorean theorem is probably the most famous theorem about triangles.
- It states that for a **right** triangle, we have

$$a^2 + b^2 = c^2,$$



where a and b are the lengths of the two sides meeting at a right angle, and c is the length of the hypotenuse.

- Examples of right triangle side lengths:

Verify: $3^2+4^2=5^2$

$9+16=25$ ✓

$8^2+15^2=17^2$ ✓

64 225 289

8, 15, 17 right triangle.

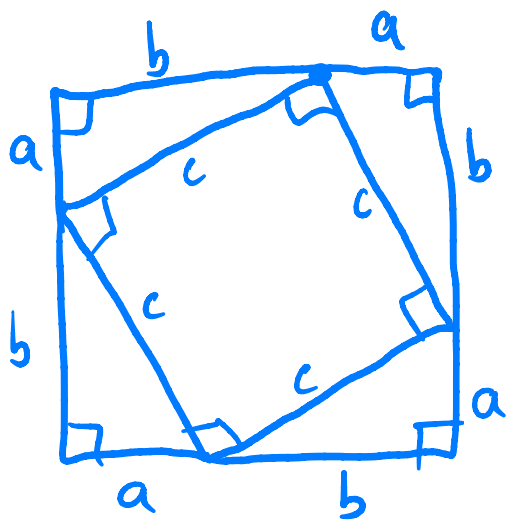
In general, if $m > n$ are any positive numbers, then $(m^2 + n^2)$, $(m^2 - n^2)$, and $2mn$ form the side lengths of a right triangle.

Ex: $m=4, n=1$: $m^2+n^2 = 16+1=17$ $m^2-n^2=16-1=15$ $2mn=2\cdot4\cdot1=8$

Proof of the Pythagorean theorem

We will prove the Pythagorean theorem using the following diagram, combined with the area formulas for triangles and squares.

2 different ways:



Calculate area of big square:

$(a+b)^2 = c^2 + 4\left(\frac{1}{2}b \cdot a\right)$

one way | area of small square | area of right triangle

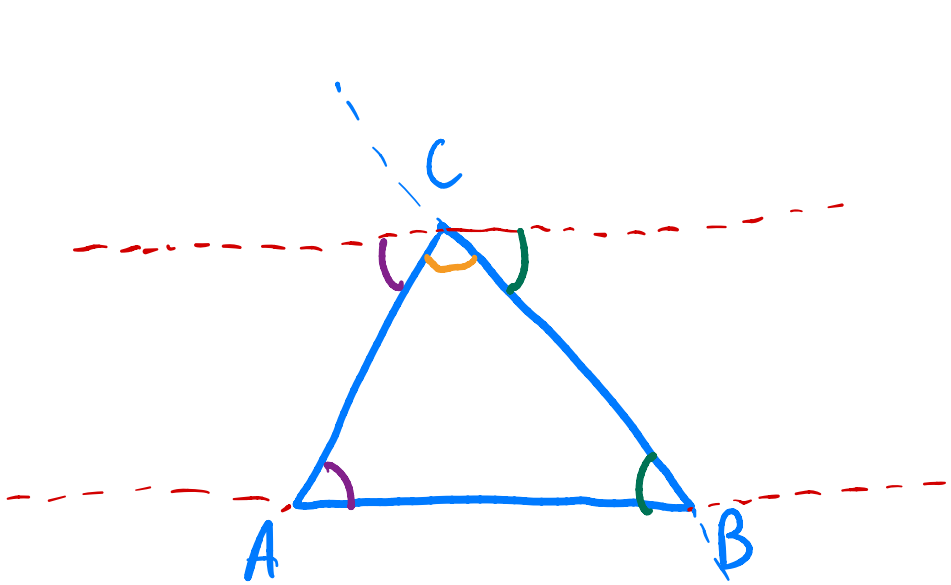
$a^2 + \cancel{2ab} + b^2 = c^2 + \cancel{2ab}$

$-2ab$ $-2ab$

So $\boxed{a^2 + b^2 = c^2}$

Angle sum of a triangle

The angles in a triangle add up to 180 degrees (or, as Euclid would say, “two right angles”), as can be seen by the diagram below.



Purple angles are equal.
Green angles are equal.

Sum of angles in the triangle is
purple + orange + green = straight line = 180°

Here we use that “alternate interior angles” have equal measure.

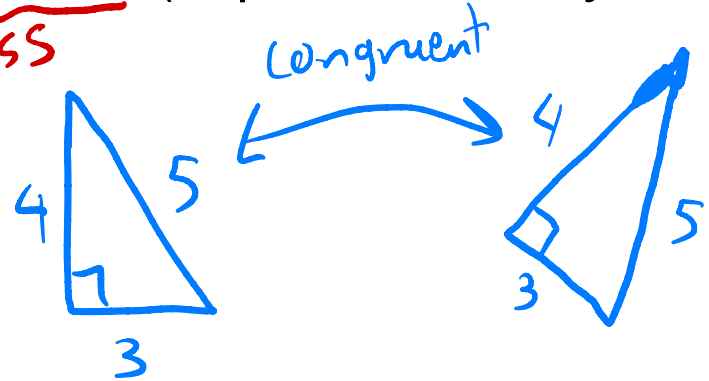
Congruent triangles

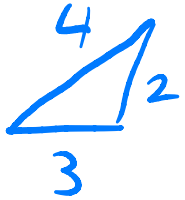
- Two triangles are *congruent* if one can be superimposed on the other by moving and rotating it. Equivalently, two triangles are congruent if the side lengths of one equal the side lengths of the other, and the angle measures of one equal the angle measures of the other.
- In order to show that two triangles are congruent, it suffices to show any of the following:
  - SSS, SAS, ASA, SAA

If 3 sides of one triangle equal the 3 sides of another triangle, then they are congruent.

- So, for instance, any two triangles with side lengths 3, 4, and 5 are congruent (in particular, any such triangle must be a right triangle).

by SSS





Not a right triangle.

Homework Exercises

1. A right triangle has two side lengths 3 and 5. What are the possible lengths for the third side? Hint: There is more than one possibility!
2. A triangle has side lengths 4, 6, and 9. Another triangle is similar to this one and has side lengths 8 and 12. What are the possible side lengths for the third side? Hint: There is more than one possibility!

2) A triangle has side lengths 7 and 13. What are all the possible side lengths for the third side?

Thank you for your attention! Next week, we will discuss more about triangles and possibly quadrilaterals.

↳ mostly triangles-