Liberian Mathematics Teacher Training Program 2023–2024

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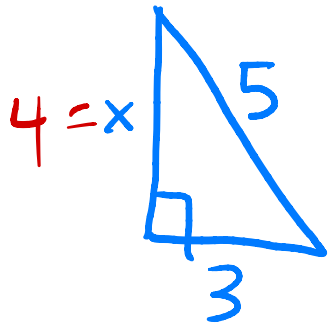
HW Exercise 1

A right triangle has two side lengths 3 and 5. What are the possible lengths for the third side? Hint: There is more than one possibility!

First Answer: 4



$3^2 + x^2 = 5^2$

$9 \quad\quad 25$

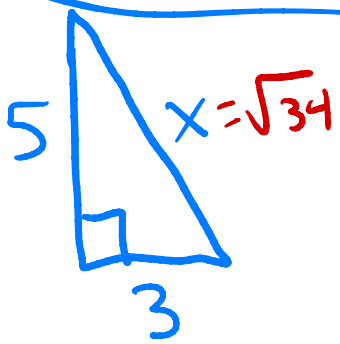
$-9 \quad\quad -9$

$x^2 = 25 - 9 = 16$

$\rightarrow \boxed{x = 4}$

Second answer:

$\sqrt{34}$



$3^2 + 5^2 = x^2$

$\rightsquigarrow 9 + 25 = x^2 \rightsquigarrow 34 = x^2 \rightsquigarrow \boxed{x = \sqrt{34}}$

HW Exercise 2

A triangle has side lengths 7 and 13. What are all the possibilities for the length of the third side?

Let x = third side.

What are the possible values of x that do not violate the triangle inequality?

The possible values are those satisfying $6 < x < 20$.

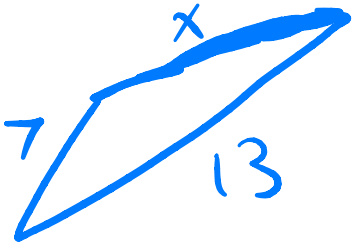
Need: ~~$x \geq 6$~~?

$x > 6$

$7 + x > 13$

$-7 \quad -7$

$x > 6$.



$x < 7 + 13$

$x < 20$

$x + 13 > 7$

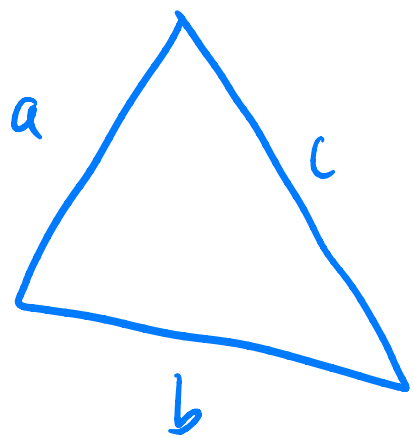
$-13 \quad -13$

$x > -6$

Automatic

Recall: The triangle inequality

The *triangle inequality* says that, in any triangle, the sum of the lengths of any two sides is always greater than the length of the third side.



$a+b>c$

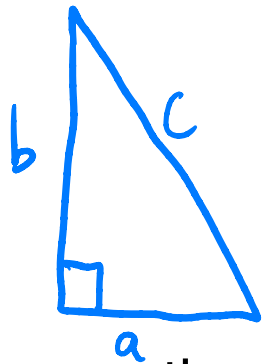
$a+c>b$

$b+c>a$

Recall: The Pythagorean theorem

- The Pythagorean theorem is probably the most famous theorem about triangles.
- It states that for a **right** triangle, we have

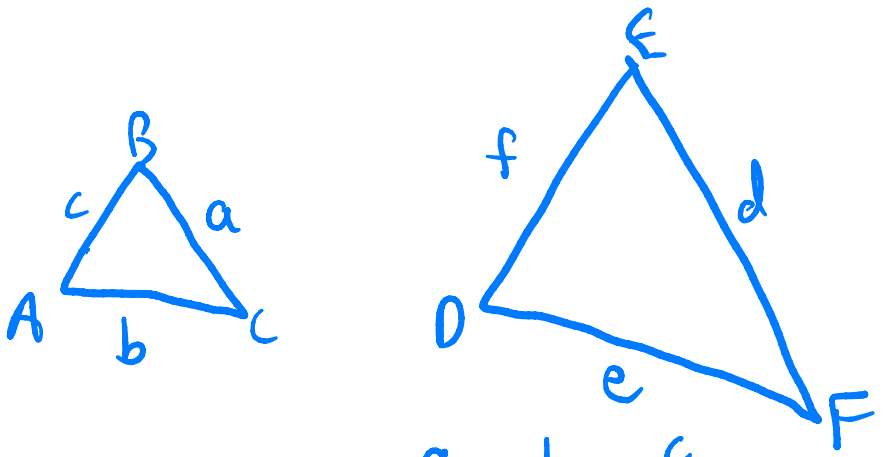
$$a^2 + b^2 = c^2,$$



where a and b are the lengths of the two sides meeting at a right angle, and c is the length of the hypotenuse.

Similar triangles

- Two triangles are *similar* if the angle measures of one equal the angle measures of the other. In this case, one triangle is a dilated (or "scaled up") version of the other one.
- In fact, it suffices to check two angle measures rather than all three. Why? $\angle C = 180^\circ - \angle A - \angle B = 180^\circ - \angle D - \angle E = \angle F$
- Similar triangles have a common ratio. So for example, we can solve for the side length in the following diagram:

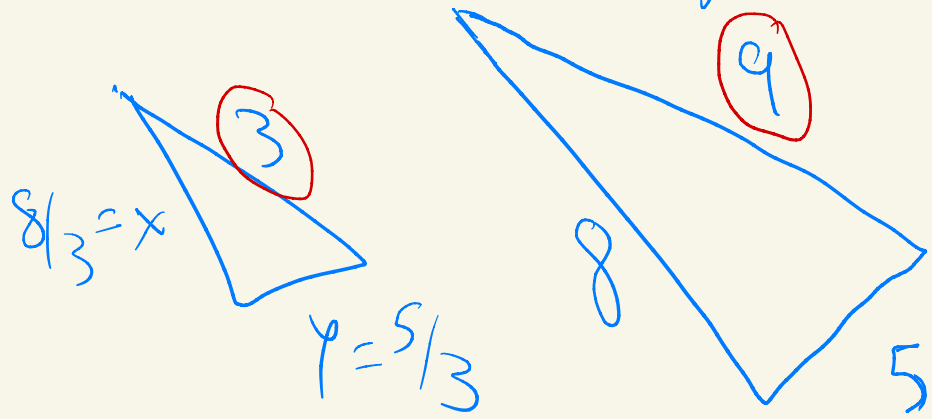


$\angle A = \angle D$, $\angle B = \angle E$, $\angle C = \angle F$

If I know $\angle A = \angle D$, $\angle B = \angle E$, then why do I automatically get $\angle C = \angle F$?

Common Ratio: $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ (True for similar triangles).

Given two similar triangles:



Q: Calculate x and y.

$3 = \frac{9}{3} = \frac{8}{x} = \frac{5}{y}$

$3 = \frac{8}{x}$

$\rightarrow 3x = 8$

$\rightarrow x = 8/3$

3 8/3

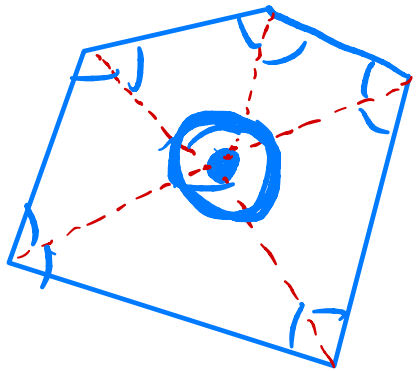
$\frac{8}{8/3} = \frac{5}{y}$

$3 = \frac{5}{y} \rightarrow 3y = 5 \rightarrow y = 5/3$

Total angle measure of an n-sided polygon

- Let $n \geq 3$, and consider a polygon with n sides.
- How can we calculate the total sum of the angles in such a polygon?
- Let's begin with the example of a pentagon.

Know: Angles of a triangle add to 180°.



splits pentagon into 5 triangles.
5 triangles total 180° each, so adding up all angles gives $180° \times 5 = 900°$.

Of this 900°, 360° are in the interior.
What's left is $900° - 360° = \boxed{540°}$ ← sum of the pentagon angles.

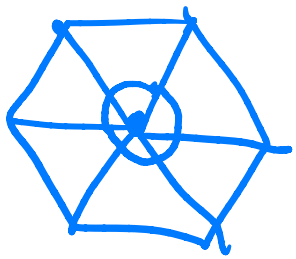
Total angle measure of an n-sided polygon, continued

Now, instead of a pentagon, suppose we have an n-sided polygon. If we take a point in the center and connect it to each vertex, how many triangles does this divide the n-gon into?

↳ n triangles!

So, the sum of the measures of the angles in an n-gon is . . .

Draw all angles and add like before!



Get $\underbrace{180^\circ \times n}_{\text{total sum}} - \underbrace{360^\circ}_{\text{interior circle}} = \boxed{180^\circ(n-2)}$

Ex: $n=3 \rightsquigarrow 180^\circ \times 1 = 180^\circ$

$n=4 \rightsquigarrow 180^\circ \times 2 = 360^\circ$

$n=5 \rightsquigarrow 180^\circ \times 3 = 540^\circ \ldots$

Formula:

Sum of interior angles of n-sided polygon is $180^\circ \times (n-2)$.

Angle measure of a regular *n*-gon

A *regular* *n*-gon is a polygon with *n* sides such that all of its sides are equal and all of its angles are equal.

From the previous slide, we calculate that the measure of each angle in a regular *n*-gon is...

Total angle measure of regular n-gon $= 180^\circ(n-2)$.

Each angle measures $\frac{180^\circ(n-2)}{n} = \frac{180^\circ \times n}{n} - \frac{180^\circ \times 2}{n}$

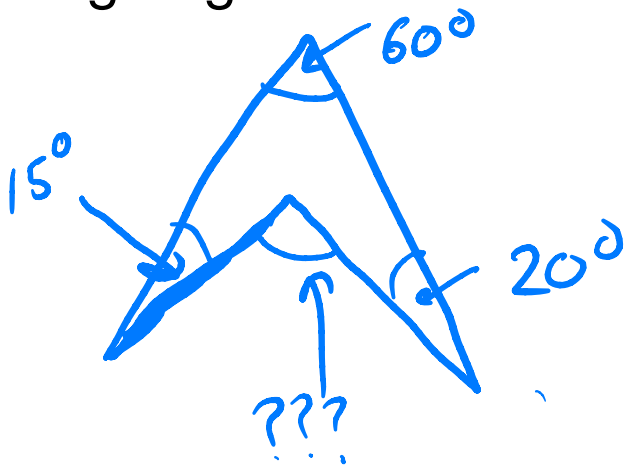
$= \boxed{180^\circ - \frac{360^\circ}{n}}$.

Example: $n=5$: $180^\circ - \frac{360^\circ}{5}$

$= 180^\circ - 72^\circ = \boxed{108^\circ}$

Homework Exercises

1. A triangle has side lengths 4, 6, and 9. Another triangle is similar to this one and has side lengths 8 and 12. What are the possible side lengths for the third side? Hint: There is more than one possibility! (2 possibilities)
2. Calculate the missing angle in the following diagram.



Thank you for your attention! Next week, we will discuss quadrilaterals.

(and do Pythagorean theorem proof using similar triangles)