Liberian Mathematics Teacher Training Program 2023–2024

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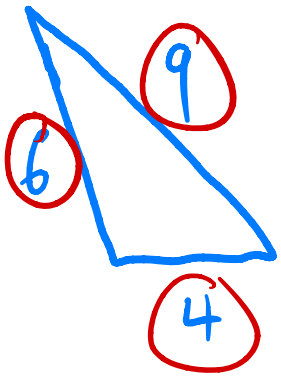
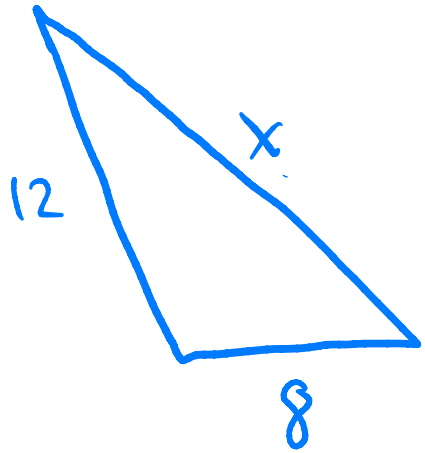
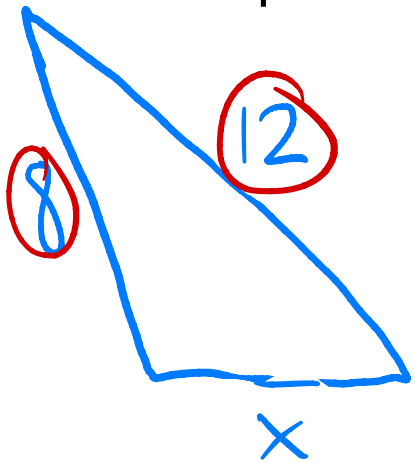
HW Exercise 1

A triangle has side lengths 4, 6, and 9. Another triangle is similar to this one and has side lengths 8 and 12. What are the possible side lengths for the third side? Hint: There is more than one possibility!







Recall: Similar triangles have a common ratio.

$$2 = \frac{8}{4} = \frac{12}{6} = \frac{x}{9} \rightsquigarrow \boxed{x = 18}$$

$$2 = \frac{x}{9} \rightsquigarrow 18 = x.$$

$$\frac{8}{6} = \frac{12}{9} = \frac{x}{4}$$

$$\rightsquigarrow 12 \cdot 4 = x \cdot 9$$

$$48 = 9x \rightsquigarrow x = \frac{48}{9} = \boxed{\frac{16}{3}}$$

HW Exercise 2

Calculate the missing angle in the following diagram.

Know: Angles in quadrilateral add to 360°

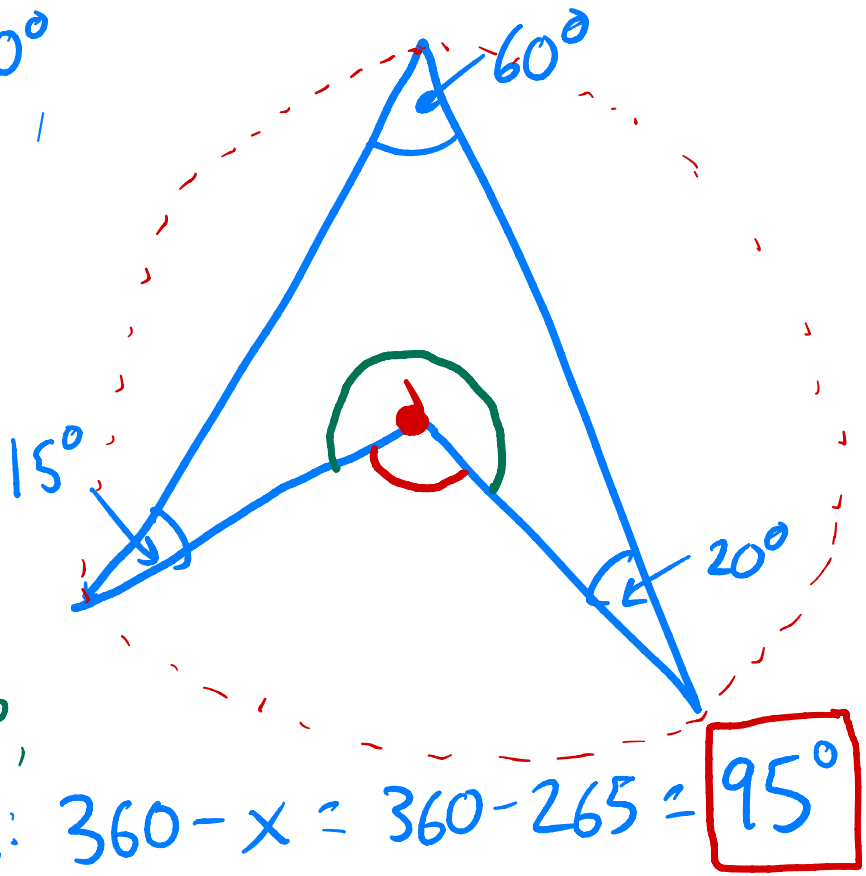
Green angle $= x^\circ$

$15 + 60 + 20 + x = 360$

$\rightarrow 95 + x = 360$

$\rightarrow x = 360 - 95$

$= 265$



If green angle is x°,

then red angle is: $360 - x = 360 - 265 = \boxed{95^\circ}$

120°.

(red + green $= 360^\circ$)

Quadrilaterals

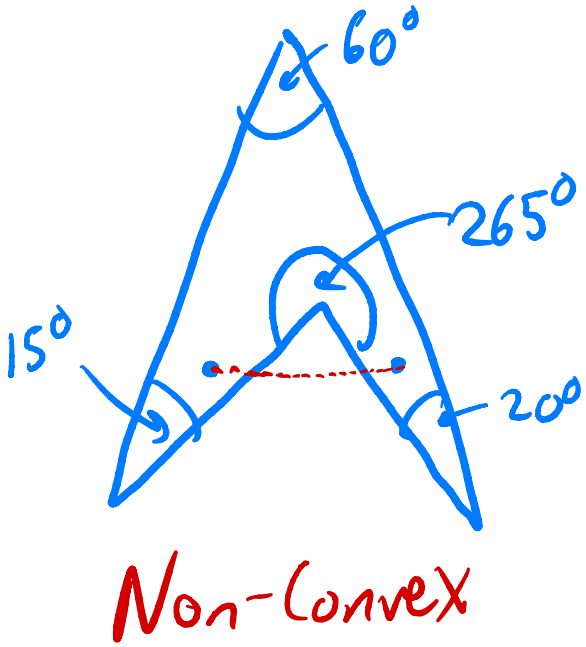
- For the past two weeks, we have studied triangles, which are polygons with 3 sides.
- Today, we will discuss *quadrilaterals*, which are polygons with four sides.
- There are lots of different types of special quadrilaterals, and lots of overlap between the types.
- For instance, unlike in the case of triangles, having all sides of a quadrilateral being equal does not guarantee that all angles are equal.
- Similarly, having all angles equal does not guarantee that all sides are equal.

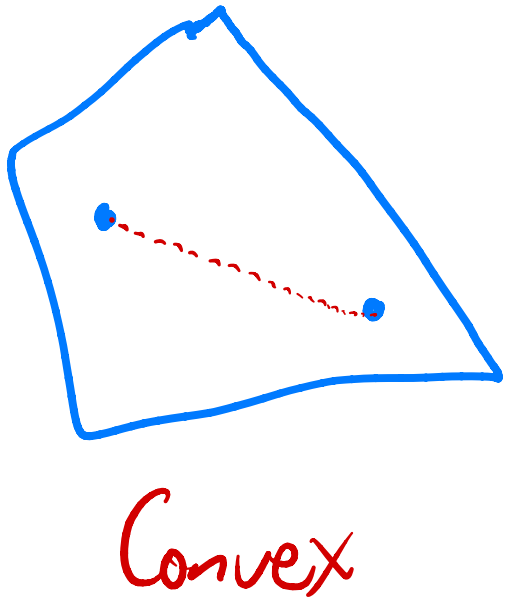
Types of quadrilaterals

- A **square** is a quadrilateral with all four sides equal and all four angles equal. (all angles are right angles).
- A **rectangle** is a quadrilateral with all four *angles* equal (so a square is a rectangle!)
- A **rhombus** is a quadrilateral with all four *sides* equal (so a square is also a rhombus!)
- A **parallelogram** is a quadrilateral with two pairs of parallel sides (in this case, both parallel sides in a pair have equal length). All rhombi and rectangles are parallelograms.
- A **trapezoid** is a quadrilateral with at least one pair of parallel sides. So all parallelograms are trapezoids.
- A quadrilateral that does not fit into any of these categories is often simply called a quadrilateral (some sources use “kite”).

Types of quadrilaterals, continued

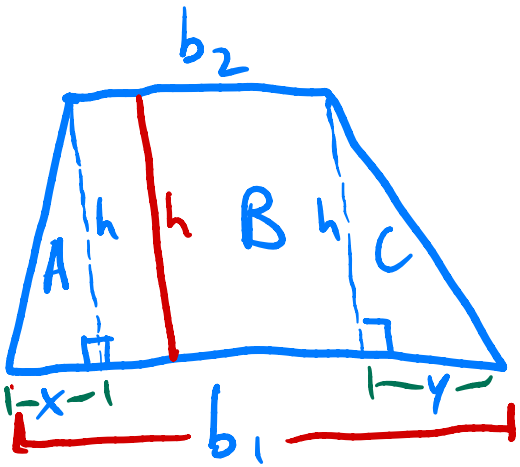
- Unlike a triangle, a quadrilateral can have an angle of over 180°!
- A quadrilateral with all angles less than 180° is called *convex*.





Area formulas for special quadrilaterals

- A square with side length s: $\text{Area} = s^2$
- A rectangle with length ℓ and width w: $\text{Area} = \ell \cdot w$
- A parallelogram with base length b and height h: $\text{Area} = bh$
- A trapezoid with parallel sides of lengths b_1 and b_2 and height h:



$$\text{Area} = \frac{b_1+b_2}{2} \cdot h$$

$b_1 = x + b_2 + y$

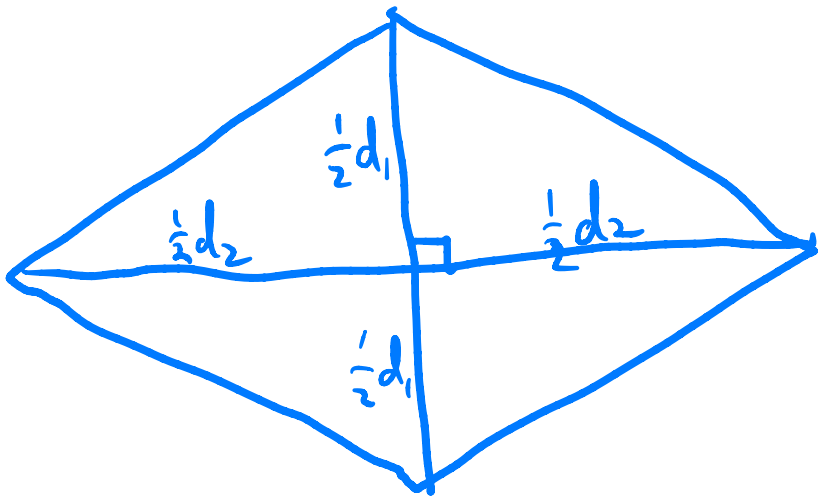
Proof: $\text{Area} = A + B + C = \frac{1}{2}xh + b_2h + \frac{1}{2}yh = \left(\frac{1}{2}x + \frac{1}{2}y + b_2\right)h$

$= \left(\frac{1}{2}(x+y+b_2) + \frac{1}{2}b_2\right)h$

$= \left(\frac{1}{2}b_1 + \frac{1}{2}b_2\right)h =$

Alternate area formula: Rhombus

Note that a rhombus has *perpendicular* diagonals. Suppose they have lengths d_1 and d_2.

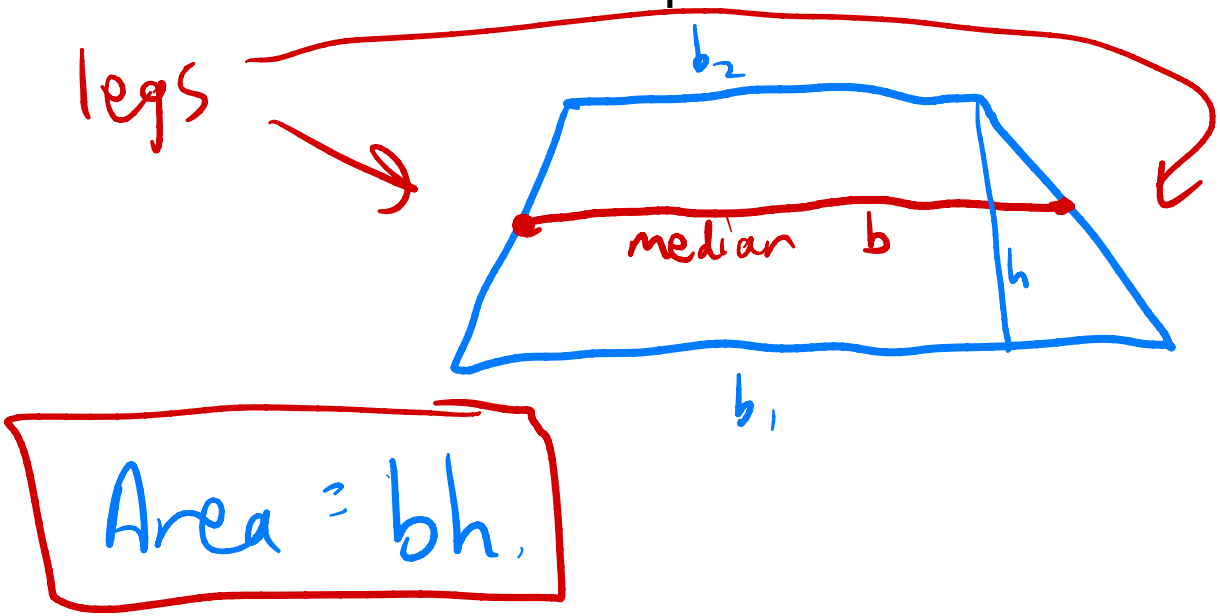


Area = Sum of 4 triangle areas = 4 · one triangle area

$$= 4 \cdot \frac{1}{2}\left(\underbrace{\frac{1}{2}d_2}_{b}\right)\left(\underbrace{\frac{1}{2}d_1}_{h}\right) = \boxed{\frac{1}{2}d_1 d_2}$$

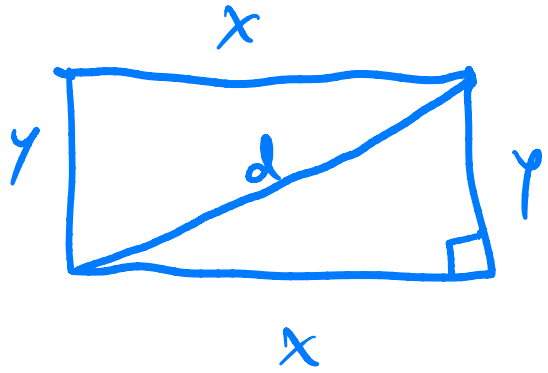
Alternate area formula: Trapezoid

- Given a trapezoid, fix two parallel sides with lengths b_1 and b_2. The other two sides are called the *legs*.
- The line segment connecting the midpoints of the two legs is called the *median*.
- If b is the length of the median, then $b = (b_1 + b_2)/2$.
- So the area of the trapezoid can be written as:



An exercise

A rectangle has area equal to 40 and a diagonal of length 10. What is its perimeter? Hint: Let x and y be the lengths of two adjacent sides of the rectangle.



Perimeter $= x+y+x+y$

$= 2x+2y$

$= 2(x+y)$

Given: Area $= 40 = xy$

Diagonal $= d = 10$. Pythagoras: $d^2 = x^2+y^2$.

Know: $40 = xy$ $\quad 10^2 = x^2+y^2$ OR $100 = x^2+y^2$

$= (x+y)^2 - 2xy$

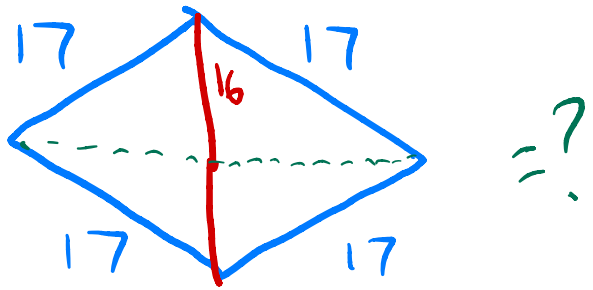
$= (x+y)^2 - 2\cdot 40$

$\rightsquigarrow 180 = (x+y)^2 \rightarrow \sqrt{180} = x+y \rightarrow 2\sqrt{180} = 2(x+y) = P$

$= 12\sqrt{5}$

A second exercise

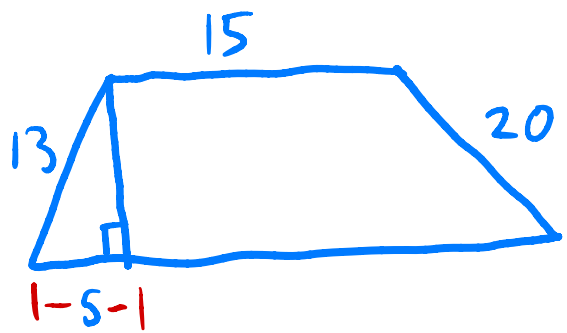
A diagonal of a rhombus has length 16. Each side of the rhombus has length 17. Find the length of the other diagonal.



Exercise. (#4)

Homework Exercises

1. The diagonal of a rectangle has length 37. The length is 1 less than 3 times the width. What is its perimeter? *Hint:* Let x be the width. Then set up an algebraic equation for x. Once you know x, then you know the length as well, and can figure out the perimeter.
2. Find the area of a rhombus with sides of length 13 and one diagonal of length 24. (use diagonal area formula)
3. Find the area of the following trapezoid:



← Multi-step (challenge)

Thank you for your attention! There will be NO CLASS next week (December 29). On January 5, we will discuss general polygons.

Merry Christmas
and Happy New Year!