Liberian Mathematics Teacher Training Program 2024–2025

Andrew Obus[1]

CUNY Baruch College

September 13, 2024

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About my work with I-Help Liberia

- My inspiration for working in Liberia is Mr. Asumana Jabateh Randolph, whom I met at Hunter College High School in New York City.
- I have helped run 3 STEM workshops in Liberia (Summer 2011, 2013, 2017). The workshops have taken place in Monrovia, Kakata, Suakoko, and most recently Ganta.
- This is the fourth year of the I-Help Liberia Mathematics Teacher Training Program, which is planned to run for a total of 5 years.
- I have also helped organize “Math Olympics” contests for high school students in Liberia in Summer 2013, 2017, and 2018.
- Over the last four years, Liberian High School students have instead participated in the Wits Mathematics Competition, which is run out of South Africa, and open to all African participants.

About my career

- I received my Bachelor's Degree in mathematics from Harvard College in 2003, and my Ph. D. in mathematics from the University of Pennsylvania in 2009.
- I am currently an Associate Professor of mathematics at Baruch College, one of the senior colleges of the City University of New York (CUNY). I do research in Number Theory and Algebraic Geometry.
- Before coming to Baruch, I taught at Columbia University and the University of Virginia.

This semester

- Our focus for this year will be **calculus**, as well as preliminary material leading up to calculus. Several people requested this material in the feedback I solicited last year.
- This material will be a little bit more *advanced* than what we have studied in the previous two years. But it is among the most important material that your students will learn.
- Let us take a break now to go over the syllabus.

I-HELP Liberia Mathematics Teacher Training Program: Fall 2024

Instructor: Dr. Andrew Obus, CUNY Baruch College, New York City, USA

Meeting time: Fridays 4:00 - 6:00 PM, unless otherwise noted. Participants who complete the entire training will receive a certificate to that effect.

Meeting place: The Zoom link for this course is `https://baruch.zoom.us/j/8820220889`

The password is `3.14159`

NEW! Website and YouTube Channel: There is now a website for this course at

`https://blogs.baruch.cuny.edu/aobus/liberian-mathematics-teacher-training-program`

where I will post notes and course materials. I also have a YouTube channel where I will post recordings of the lectures, located at

`https://www.youtube.com/@andrewobus747/videos`.

Chat room: There is a Whatsapp chatroom for this course entitled "Math Teacher Training". Please contact Sangay Freeman to be added to this chat room. I will post important notices to this chat room from time to time.

Textbook: I will draw inspiration from the textbook "Single Variable Essential Calculus, 2nd Edition" by James Stewart.

Objectives: The theme of this semester will be **calculus**, specifically **differential calculus**. Unlike some of the other topics we have worked on in previous years, this may be a topic that is completely new to many of you. We will continue studying differential calculus in the spring. Depending on the interests of the class, we may continue on to study some **integral calculus**, or we may focus more on **sequences and series**.

In order to build the foundation to study calculus, we will need to spend time at the beginning of the term learning about *relations*, *functions*, and *limits*.

Homework: Practice exercises will be assigned at the end of every session. Working on these problems is *essential* if you want to get the most out of the training. Participants will present and discuss solutions at the next session.

Attendance Policy: Participants are expected to attend classes *consistently*. This training is cumulative (even more so than in previous years!), and the sessions build on each other. It will be very difficult to follow the course when you miss a session. If you miss more than 3 sessions, you may be dropped from receiving data funds for the training.

In particular, the material for Spring 2025 will depend heavily on the material from Fall 2024. Thus, **only participants who attend consistently in Fall 2024** will be given funding for the program in Spring 2025. No new participants will be funded. If you are unable to attend consistently in Fall 2024, there will be another year of the program in Fall 2025 – Spring 2026 which you may join.

Schedule of topics:

Date	Topic
September 13	Introductions, what is calculus?
September 20	Relations and functions
September 27	Review of the standard functions
October 4	NO CLASS
October 11	Limits of functions, the finite case
October 18	NO CLASS
October 25	Limits of functions, the infinite case
November 1	Left and right limits, continuity
November 8	NO CLASS
November 15	The meaning of the derivative
November 22	Computing simple derivatives
November 29	NO CLASS
December 6	APPLICATION: Motion and physics
December 13	Rules for computing derivatives
December 20	APPLICATION: Maxima, minima, and optimization problems

This semester, continued

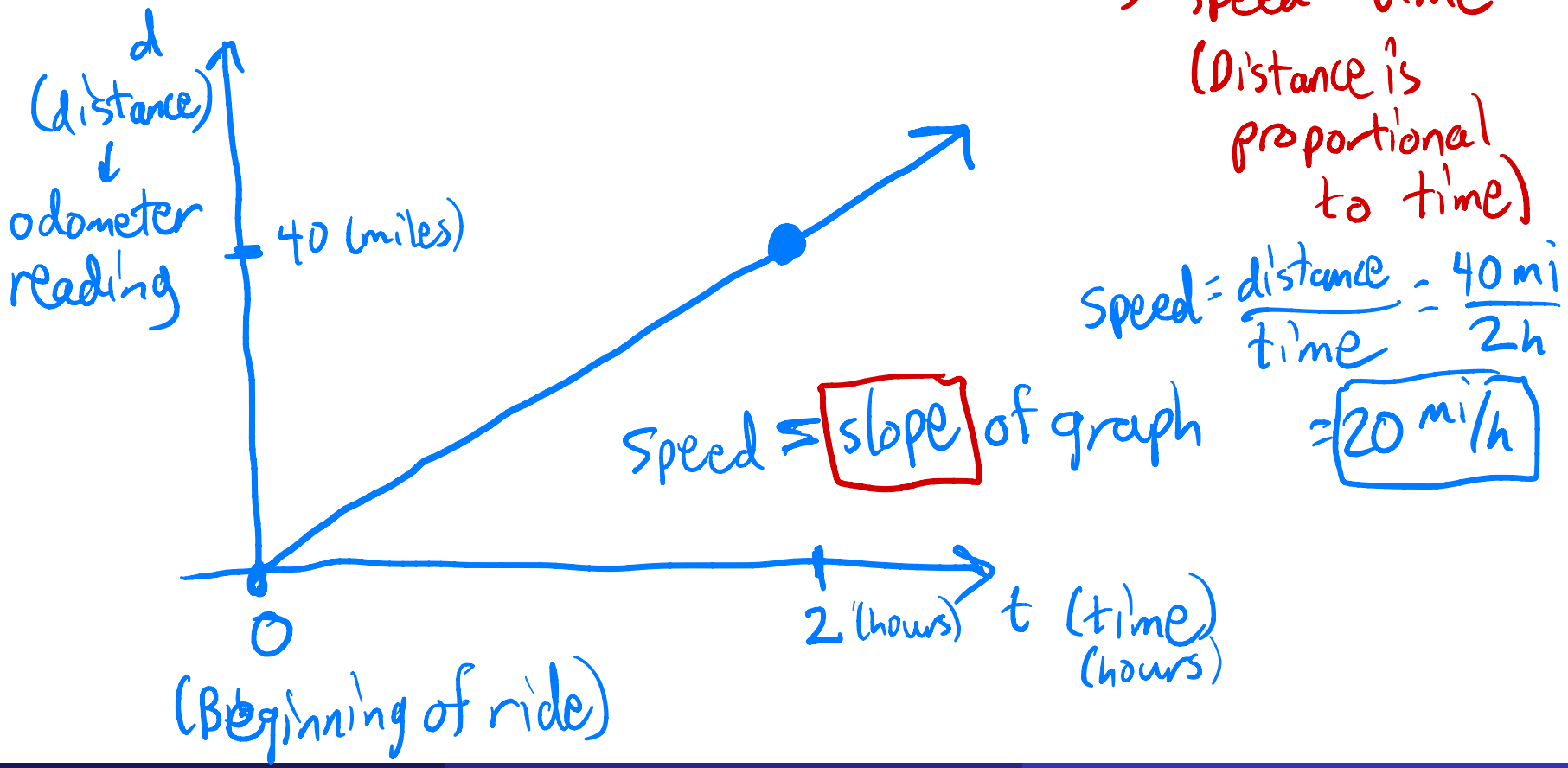
- The training now has a website:
  `https://blogs.baruch.cuny.edu/aobus/liberian-mathematics-teacher-training-program/`
  It has not been updated since last year — I will update it next week.
- I will post lecture notes and video recordings on this website. I will also post them to the WhatsApp chat room.
- If you forget the web address, you can put “Andrew Obus Baruch” into Google to get my main website, and then there is a link to the Liberia program website at the bottom of the page.
- Starting next week, there will be homework problems given out after every meeting. Attempting and discussing the homework is essential for you to get the most out of the program.
- We will usually discuss the homework at the beginning of each class.

What is calculus?

- Calculus is the mathematics of **motion** and **change**.
- Furthermore, calculus is used to extrapolate facts we know about **straight** objects to the case of **curved** objects.
- Lastly, calculus is about the **infinite**, both the infinitely big, and more importantly, the infinitely small (or *infinitessimal*).
- Calculus is traditionally divided into **differential calculus** and **integral calculus**.
- Many people consider calculus one of the most brilliant creations of the human mind.

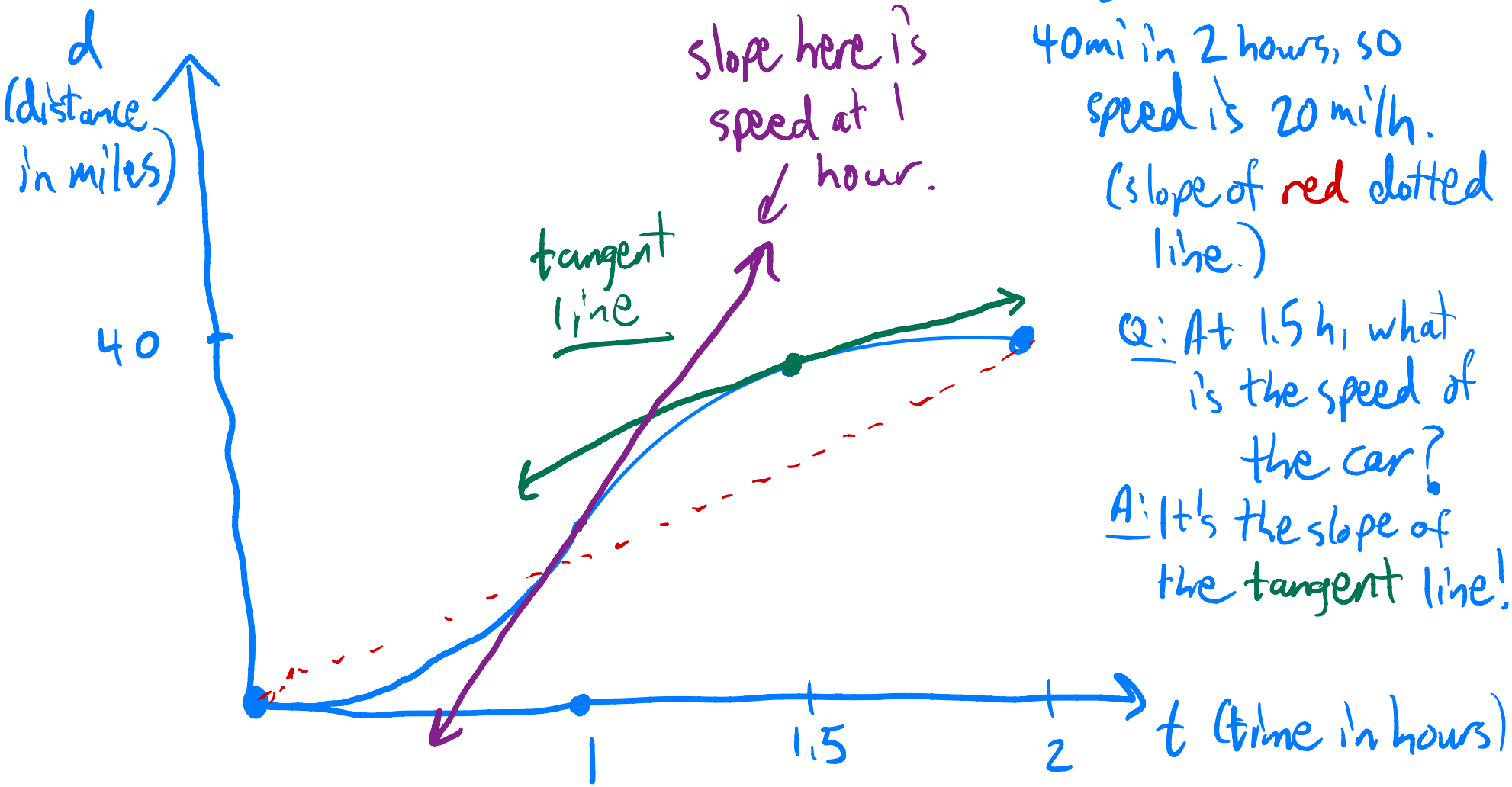
Differential calculus: The odometer problem

- Suppose a car is driving in a straight line, and we sit inside and watch the odometer. Can we determine what speed the car is going at a given time?
- If the car goes at a constant speed, this is easy.



The odometer problem, continued

What if the car speeds up/slows down?



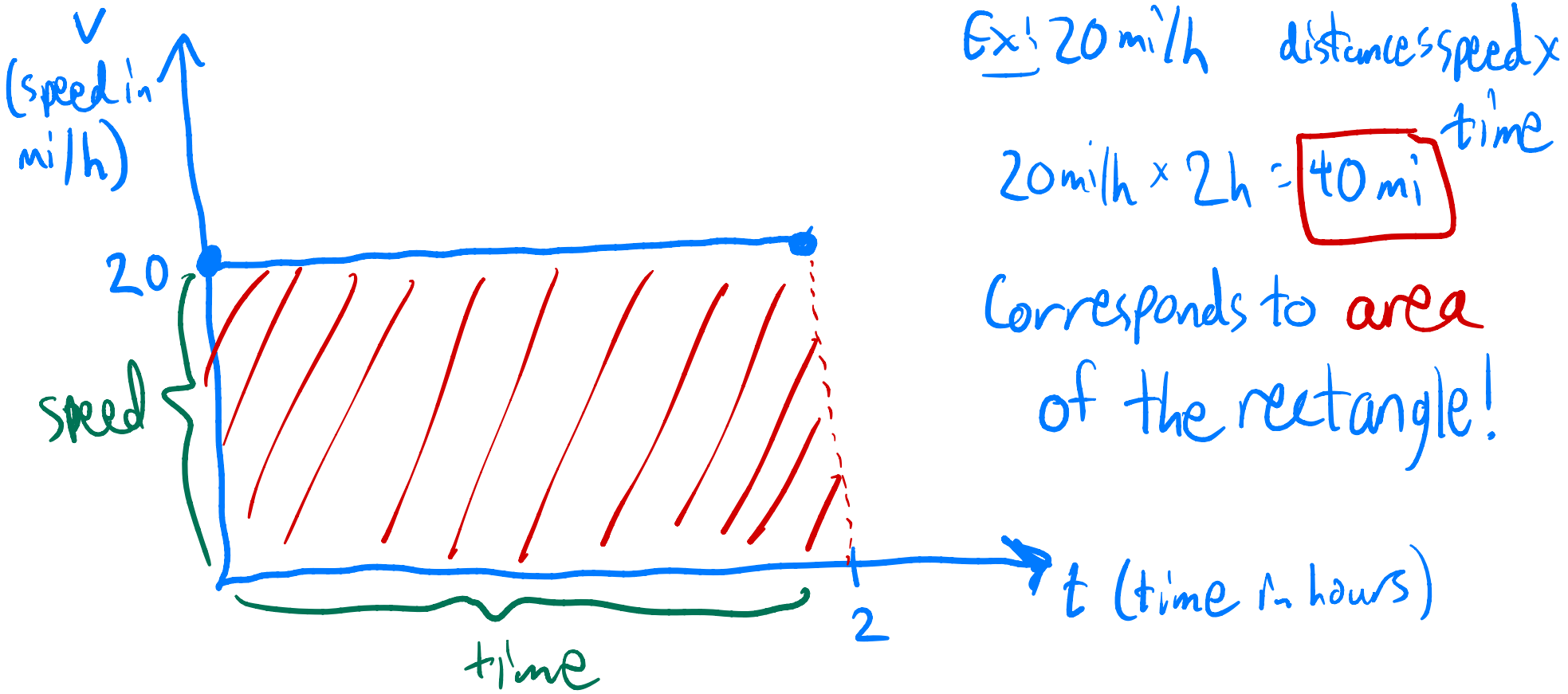
On average, car goes 40mi in 2 hours, so speed is 20 mi/h. (slope of red dotted line.)

Q: At 1.5 h, what is the speed of the car?

A: It's the slope of the tangent line!

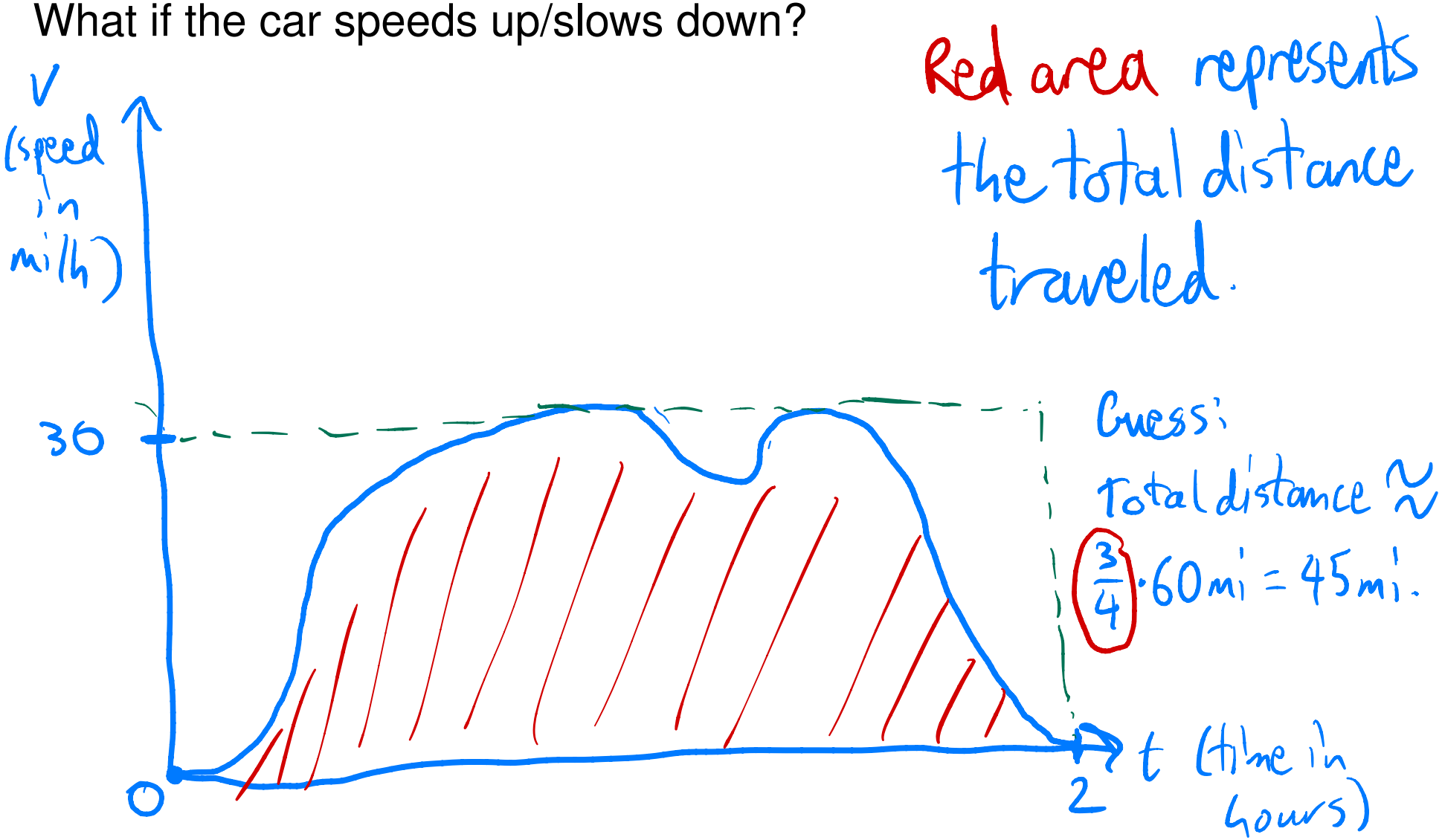
Integral calculus: The speedometer problem

- Suppose a car is driving in a straight line, and we sit inside and watch the speedometer. Can we determine how far the car has gone?
- Again, if the car goes at a constant speed, this is easy.

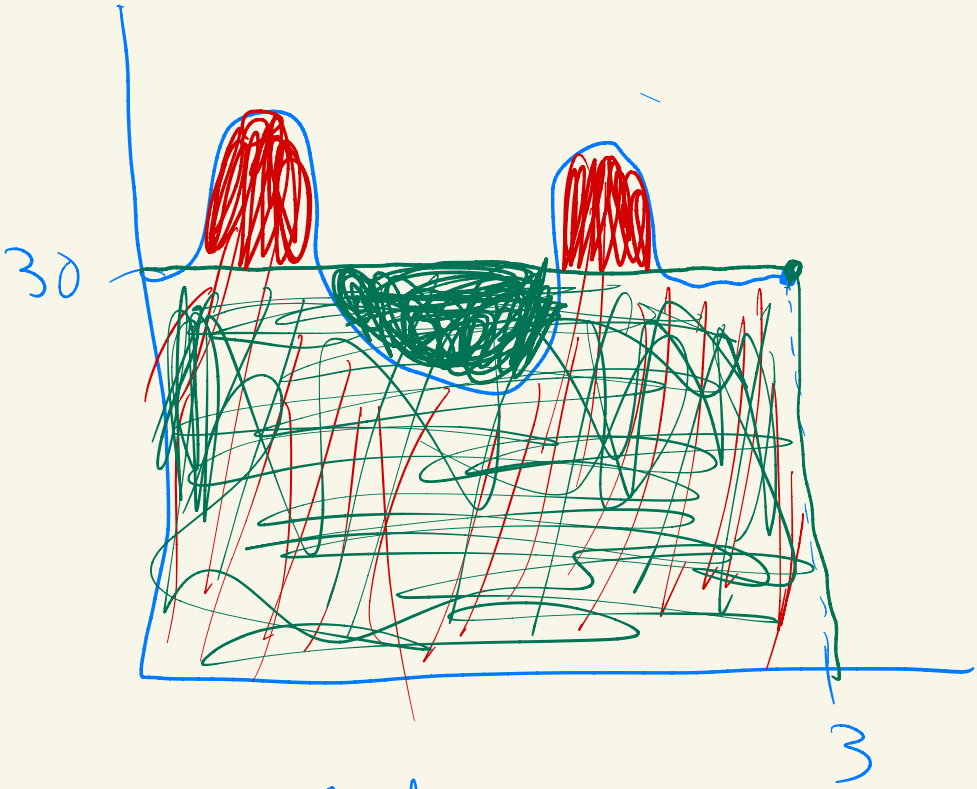


The speedometer problem, continued

What if the car speeds up/slows down?



Green Area $\approx$ Red Area



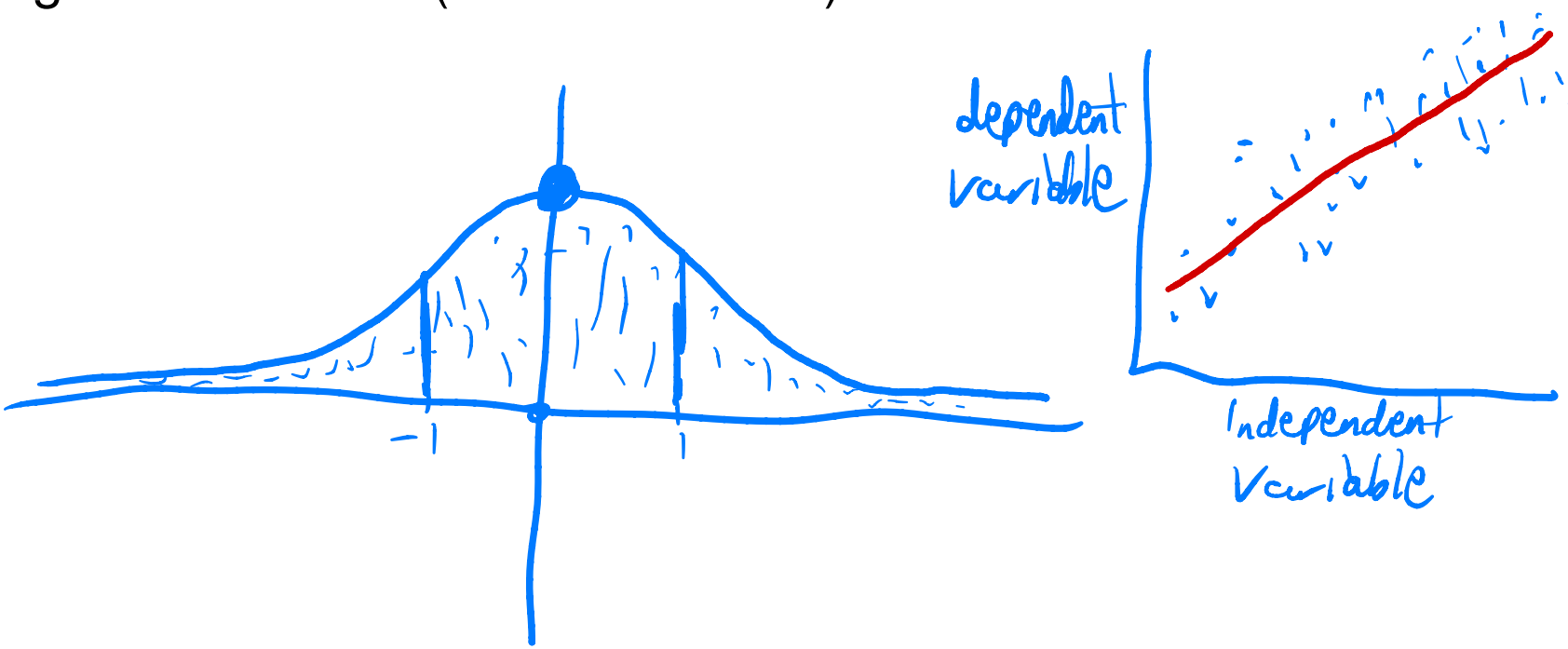
Guess: Red area $\approx 30 \times 3 = 90$.

Applications of calculus: Physics/Engineering

- Relating position, velocity, and acceleration.
- Relating work, force, and energy.
- Formulating the fundamental laws of electromagnetism.
- Relating voltage and current in capacitors.
- Much, much more. Calculus is *completely essential* to just about all aspects of modern physics.

Applications of calculus: Probability/Statistics

- Analyzing continuous probability distributions (e.g., the normal distribution, or “bell curve”).
- Relating probability density functions to cumulative distribution functions.
- Fitting “best-fit lines” (or other curves) to data.



Applications of calculus: Business/Economics/Finance

- Optimization problems: Maximizing profit/minimizing cost.
- Relating marginal (cost/valuation/profit) to total (cost/valuation/profit).
- Understanding how changing an input to an economic model affects the outputs.
- Analyzing the price of a financial instrument (say, a call option) based on its underlying parameters.

Applications of calculus: Biology/ecology

- Population dynamics in ecology.
- Predator-prey systems. ← "differential equations"
- Infectious disease modeling.
- Pharmacokinetics (the study of drug concentration in the body over time).

Applications of calculus: Geometry

- What is the area of a circle? πr^2 (r = radius)
- What is the volume of a cone? $\frac{1}{3}\pi r^2 h$
- What is the volume of a sphere? $\frac{4}{3}\pi r^3$
- How are circumference, area, and volume related?

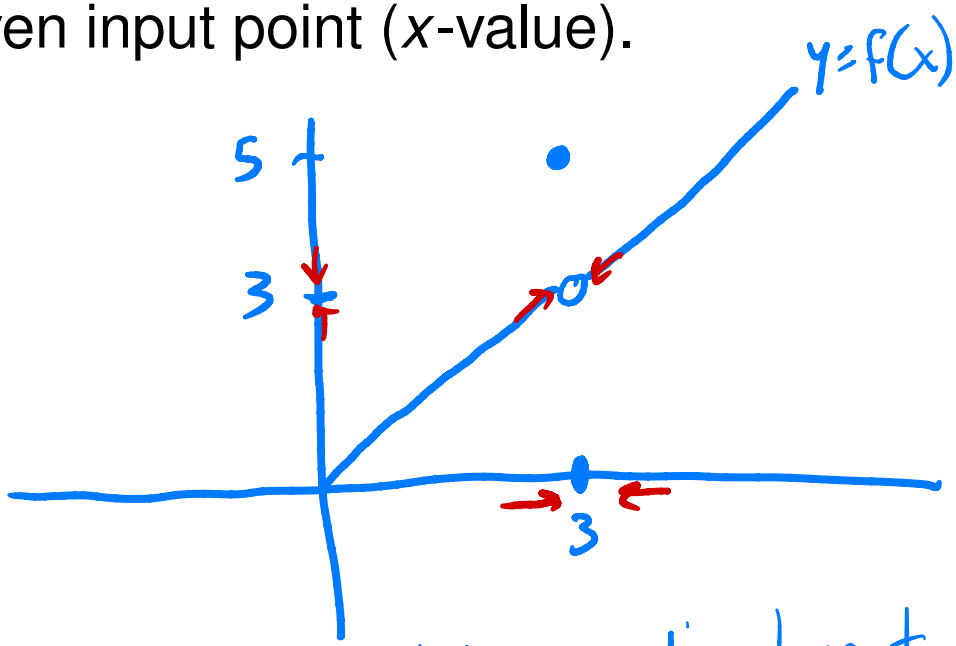
Need calculus to derive these formulas!

What do we need to know in order to do calculus?

- The most fundamental concept in calculus is that of *function*.
- Fundamentally, a function is a recipe that accepts an input (generally a real number in the context of calculus) and generates exactly one output (also a real number in the context of calculus). This output is *well-defined* (that is, the same input always gives the same output).
- We will spend quite a bit of time at the beginning of the term reviewing functions, as well as the similar concept of *relations*.
- The most important functions for us are the polynomial functions, rational functions, trigonometric functions, exponential functions, and logarithmic functions.

What do we need to know in order to do calculus?

- The next most important concept in calculus is that of *limit*.
- Defining limits rigorously is actually quite difficult! We will take a more intuitive approach this term.
- The idea of a limit is to ask how a function behaves near, but not at, a given input point (x-value).



$$f(x) = \begin{cases} x & x \neq 3 \\ 5 & x = 3 \end{cases}$$

$$\lim_{x \to 3} f(x) = 3$$

"As x gets close to 3, $f(x)$ gets close to 3"

Limit examples

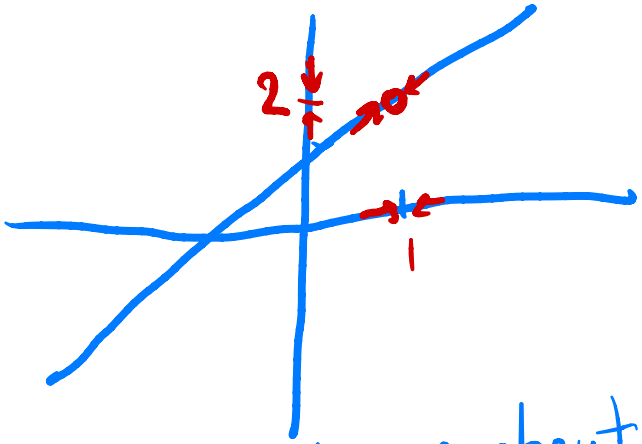
- Example: What is $\lim_{x\to 1}(x^2-1)/(x-1)$?
- Example: What is $\lim_{x\to\infty} 1/x$?

$(1^2-1)/(1-1) = 0/0$ undefined

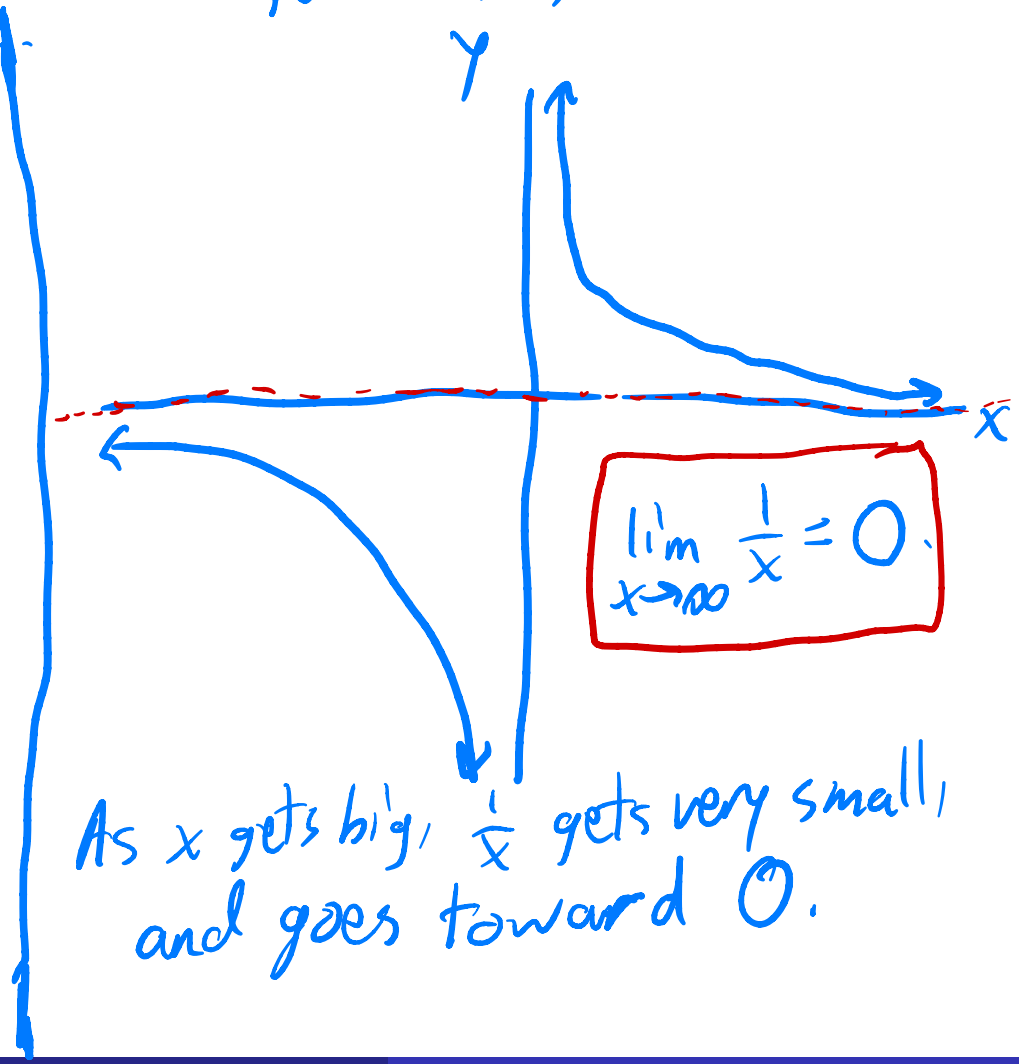
→ As x gets really big, what happens to $\frac{1}{x}$?

$$\lim_{x\to 1}\frac{x^2-1}{x-1} = \lim_{x\to 1}\frac{(x-1)(x+1)}{x-1}$$

$$= \lim_{x\to 1} x+1 = 2$$



Limit doesn't care about hole.



As x gets big, $\frac{1}{x}$ gets very small, and goes toward 0.

Thank you for your attention, and I'm looking forward to our calculus journey together!